Exercise 62

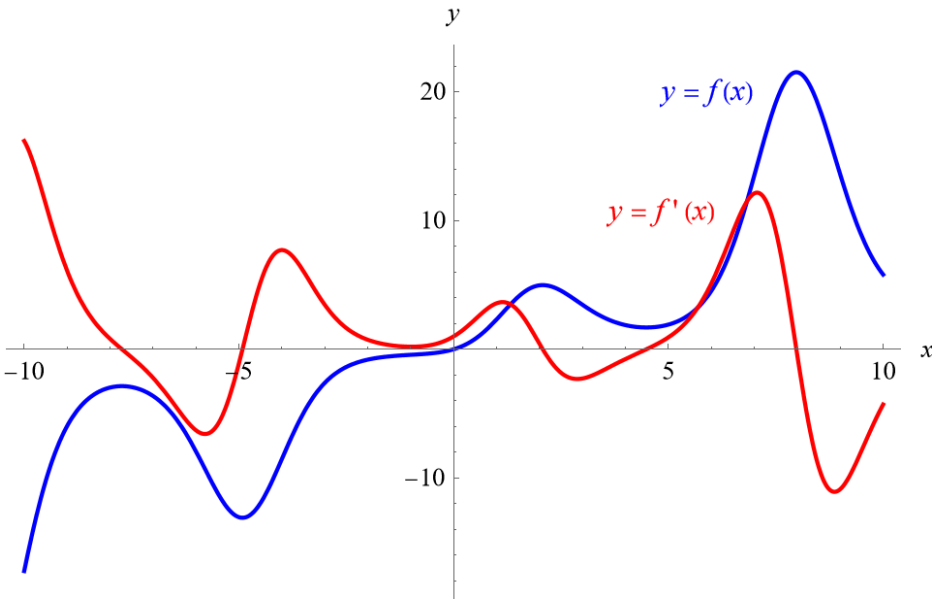
If $f(x) = xe^{\sin x}$, find $f'(x)$. Graph f and f' on the same screen and comment.

Solution

Calculate $f'(x)$ by using the chain and product rules.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(xe^{\sin x}) \\ &= \left[\frac{d}{dx}(x)\right] e^{\sin x} + x\left[\frac{d}{dx}(e^{\sin x})\right] \\ &= (1)e^{\sin x} + x\left[(e^{\sin x}) \cdot \frac{d}{dx}(\sin x)\right] \\ &= e^{\sin x} + x[(e^{\sin x}) \cdot (\cos x)] \\ &= e^{\sin x} + xe^{\sin x}\cos x \\ &= e^{\sin x}(1 + x\cos x) \end{aligned}$$

Below is a graph of $f(x)$ and $f'(x)$ versus x.



Notice that $f'(x)$ is zero whenever the tangent line to $f(x)$ is horizontal, $f'(x)$ is positive whenever $f(x)$ is increasing, and $f'(x)$ is negative whenever $f(x)$ is decreasing.